

# Theoretical Analysis of Random-Modulation Continuous Wave LIDAR

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## ABSTRACT:

A theoretical analysis of the error probability and the precision of Random Modulation-Continuous Wave LIDAR systems is presented. Both the signal to noise ratio prior and after the correlation process are calculated taking into account the main noise sources. Analytical expressions for the estimation of the probability of catastrophic errors in the determination of the distances due to random noise peaks are derived. The effect of the signal to noise ratio on the precision is also theoretically evaluated.

As illustrative examples, plots for the signal to noise ratio, the error probability and the precision corresponding to a LIDAR system currently developed in our laboratory are presented.

**Key words:** LIDAR, Random-Modulation Continuous Wave LIDAR, signal-to-noise evaluation.

## 1. - Introduction

Distance measuring is useful for many applications. Non-contact range-finding devices, which send a signal to a target and use the reflected signal to measure the distance, can be developed using acoustic or electromagnetic signals at different frequencies. Optical signals are needed when a high resolution is required, especially with small targets, giving rise to LIDAR (Light Detection and Ranging) systems.

There are three main methods of LIDAR systems for measuring distances: triangulation, interferometry and time-of-flight (ToF) [1], being the last one the most generally employed. The ToF method can be applied in pulsed and continuous wave (CW) operation (see [2] for a comparative description of the different systems). In order to reach good precision in the measurement, pulsed systems require a high peak optical power and an electrical low-pass filter with high cut-off

frequency. CW systems can solve these disadvantages.

Random-Modulation Continuous Wave (RMCW) LIDAR was proposed by Takeachi et al. [3] and it has been applied to different measurement scenarios [4-5]. Fig. 1 shows the schematics of a RMCW LIDAR system. A pseudorandom bit sequence (PRBS) is used to modulate the laser emission. The optical signal is collimated, reflected by the target and collected by the receiver optics. An optical band-pass filter rejects the undesired wavelengths to reduce the background noise. A photodetector (typically an Avalanche Photodiode APD), together with the corresponding electronics, converts the optical signal into the electrical domain. The distance is calculated from the peak of the correlation between the original sequence and received signals.

The PRBS is usually generated with an  $n$ -step shift-register which produces  $N$  ( $N = 2^n - 1$ ) bits, each one with a bit duration  $T_c$ .

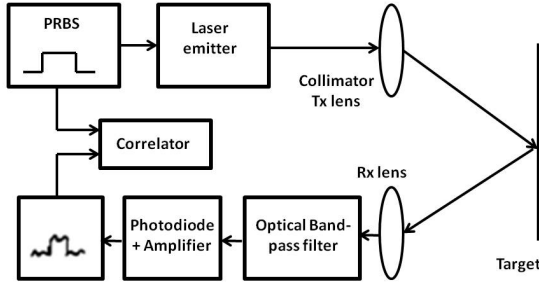


Fig. 1: Schematics of a RMCW Lidar

In this work, we provide a theoretical analysis of the error probability and precision of RMCW systems, and we apply the results to typical values of a real system currently under development.

## 2. – Signal to Noise Ratio (SNR)

### 2.1- SNR in the temporal domain

Mitev et al. [4] have evaluated the SNR of analogue RMCW lidar systems. In the following we explain the origin of the different signal and noise terms. We define first the instantaneous signal ( $S_i$ ) and noise terms ( $N_i$ ) at the temporal output of the receiver previous to the correlation process.

Considering Lambertian reflection on the target and the distance far enough, the received average signal optical power  $P_i$  is limited by the receiver lens area instead of its Field of View (FoV), and can be expressed as

$$P_i = P_o \eta_s \eta_r \alpha \frac{D_r^2}{4L^2} \quad (1)$$

where  $P_o$  is the emitted average optical power,  $\eta_s$  and  $\eta_r$  the optical efficiencies of the emitting and receiving lenses respectively,  $\alpha$  the reflectivity or albedo of the target,  $D_r$  the diameter of the receiving lens, and  $L$  the distance to the target. The average signal photocurrent  $S_i$  is given by

$$S_i = P_i \mathcal{R} M \quad (2)$$

being  $\mathcal{R}$  the photodetector responsivity and  $M$  the APD multiplication factor. The rms value of the noise  $N_i$  is given by

$$N_i = \sqrt{\sigma^2(i_s) + \sigma^2(i_b) + \sigma^2(i_d) + \sigma^2(i_{th})} \quad (3)$$

where  $\sigma(i_s), \sigma(i_b), \sigma(i_d)$  are the signal, background and dark current shot noises, respectively, and  $\sigma(i_{th})$  the thermal noise. The standard expression of the signal, dark and thermal noises are well known (see [4]), and we describe with more detail the calculation of the background noise.

If the target is far enough, the received background optical power  $P_{back}$  depends on the distance to the target and on the diameter of the receiving lens, and can be expressed as

$$P_{back} = \eta_r \alpha E_\lambda \omega \pi \frac{D_r^4}{16L^2}, \quad (4)$$

being  $E_\lambda$  the irradiance of the sun in the Earth surface and with daylight (worst case is 12 am), and  $\omega$  the bandwidth of the optical band-pass filter. Some graphical examples of  $E_\lambda$  can be found in [6,7], which is dependent on the daytime, height and geographical areas. The background shot noise is

$$\sigma^2(i_b) = 2qBFM(P_{back}\mathcal{R}M), \quad (5)$$

with  $q$  the electron charge and  $F$  the Excess Noise Factor (in case of APD). The receiver electrical bandwidth  $B$  is usually taken as  $B = \frac{1}{2T_c}$ , which is the best compromise to receive the first harmonic of PRBS with minimum noise.

### 2.2- SNR after the correlation

Figure 2 shows an example, based on numerical simulations, of the signal after the correlation. The peak indicates the time corresponding to the distance to the target, and the height of the peak is defined as the long term Signal ( $S_{LT}$ ). The long term noise  $N_{LT}$  corresponds to the rms value of the correlation function excluding the maximum.

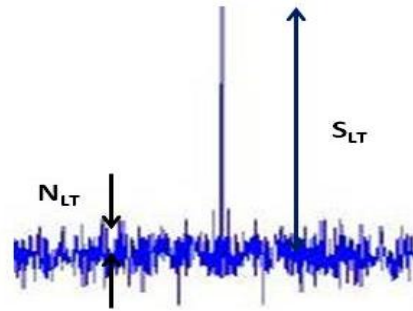


Fig. 2: Correlation function.

It can be demonstrated [4] that the long-term and instantaneous SNRs are related by:

$$\frac{S_{LT}}{N_{LT}} = \frac{S_i}{N_i} \sqrt{\frac{L_{seq}N}{4}}, \quad (6)$$

being  $L_{seq}$  the number of sequences used in the correlation.

Fig. 3 shows the ratio  $S_{LT}/N_{LT}$  as a function of the target distance for a system which is under development in our laboratory. The parameters used in the calculations are summarized in Table I. For this particular example, the predominant noise source is the dark current shot noise.

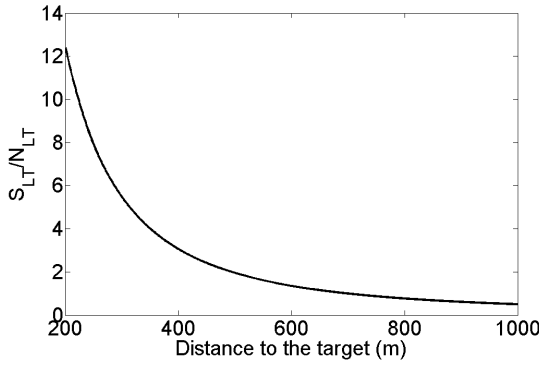


Fig 3:  $S_{LT}/N_{LT}$  as a function of target distance for the system parameters in Table 1.

Parameter	Symbol	Value
Emitter average power	$P_0$	20 mW
Number of bits in sequence	$n$	16
Bit time	$T_c$	40 ns
Receiver Lens Diameter	$D_r$	5 cm
Wavelength	$\lambda$	1.5 $\mu\text{m}$
Solar Irradiance	$E_\lambda$	0.2 W/m <sup>2</sup> nm
Reflectivity of the target	$\alpha$	0.6
Emitter Optical Efficiency	$\eta_s$	0.6
Receiver Optical Efficiency	$\eta_r$	0.6
Detector Responsivity	$\mathcal{R}$	0.9
APD multiplication factor	$M$	10
Integration Time	$L_{seq} N T_c$	10 ms
Excess Noise Factor	$F$	3
Dark Current	$i_d$	60 nA
Optical Filter Bandwidth	$\omega$	12 nm
Amplifier Noise Figure	$NF$	5 dB
Load Resistance	$R_{ld}$	5 K $\Omega$

Table I: System parameters

#### 4. – Error probability

We calculate now the probability of obtaining a catastrophic error in the distance measurement, i.e., that the maximum value of the correlation signal does not correspond to the distance to the target due to a random peak of the noise. As far as we know, this error probability has not been considered in previous works on LIDAR ranging.

A measurement error will be obtained if at least one of the samples of the correlation function, at a time different of the corresponding target distance time, takes a value higher than  $S_{LT}$ . Considering a Gaussian distribution of the noise, the probability for a sample to be higher than  $S_{LT}$ ,  $P(N > S)$  can be expressed as

$$P(N > S) = \frac{1}{2} \operatorname{erfc}\left[\frac{S_{LT}}{N_{LT}\sqrt{2}}\right] \quad (7)$$

The number of bits used in the correlation  $N_{samples}$  depends on the maximum distance to be measured  $L_{max}$  and can be expressed as:

$$N_{samples} = \frac{2L_{max}}{cT_c}, \quad (8)$$

with  $c$  the speed of the light. In consequence, the probability can be calculated as:

$$P_{er} = 1 - \left(1 - \frac{1}{2} \operatorname{erfc}\left[\frac{S_{LT}}{N_{LT}\sqrt{2}}\right]\right)^{N_{samples}} \quad (9)$$

In a real system, the analogue signal is digitalized at a sampling frequency  $1/T_c$ . In consequence, the correlated signal (see Fig. 5) will not present the ideal triangular shape and the maximum signal value  $S_{LT}$  will not be obtained. Two samples will be placed inside the ideal triangle with possible values comprised between  $S_{LT}/2$  and  $S_{LT}$ . This implies that the measurement error will be higher than that given by expression (9). We have estimated this effect by considering a discrete distribution of the position of the sampling time with respect to the maximum time.

Fig. 4 shows the calculated error probability as a function of the ratio  $S_{LT}/N_{LT}$  for a maximum ranging distance of 500 m.

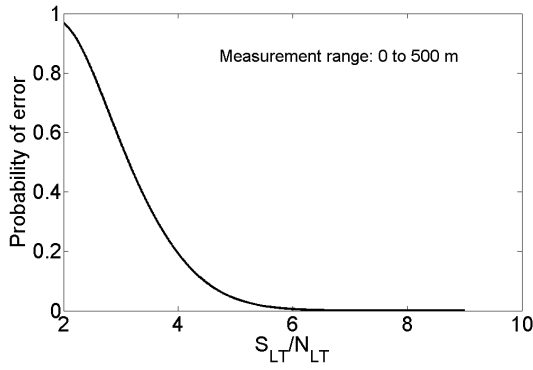


Fig. 4. Error probability as function of  $S_{LT}/N_{LT}$

## 5. – Precision of measurements

The precision of the measurements will be affected by the signal to noise ratio. In the following we estimate the precision assuming that the temporal signals are perfectly squared, i.e., a system with infinite bandwidth. In this case the correlation signal at the time corresponding to the target distance is a triangle with base  $2T_c$  and height  $S_{LT}$  (see Fig. 5). With a sampling rate equal to  $1/T_c$  two samples will be placed inside the triangle, with times (values) given by  $t_1(V_1)$  and  $t_2(V_2)$ . The real time corresponding to the target distance can be estimated from  $t_1$ ,  $V_1$  and  $S_{LT}$ , which is known from the average measured signal. Both  $V_1$  and  $S_{LT}$  are affected by the noise  $N_{LT}$ , and therefore the temporal error  $\sigma_t(1)$  for a single sample will be lower than two times the error in amplitude times the slope of the triangle  $T_c/S_{LT}$ :

$$\sigma_t(1) \leq \frac{T_c \cdot 2 \cdot N_{LT}}{S_{LT}} \quad (10)$$

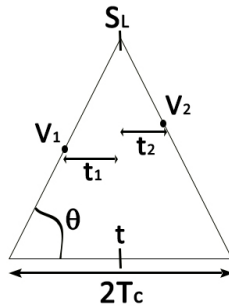


Fig. 5. Triangle formed in the correlation, sampling at our bit rate.

If we consider the two independent measurements through  $V_1$  and  $V_2$ , we obtain the error in the time  $\sigma_t$ :

$$\sigma_t \leq \frac{\sqrt{2} \cdot T_c}{S_{LT}/N_{LT}}, \quad (11)$$

which can be converted into error in distance  $\sigma_d$ , yielding:

$$\sigma_d = \frac{\sigma_t c}{2} \leq \frac{\sqrt{2} T_c c}{2 S_{LT}/N_{LT}} \quad (12)$$

Fig. 6 shows the precision of the measurements as a function of  $S_{LT}/N_{LT}$ .

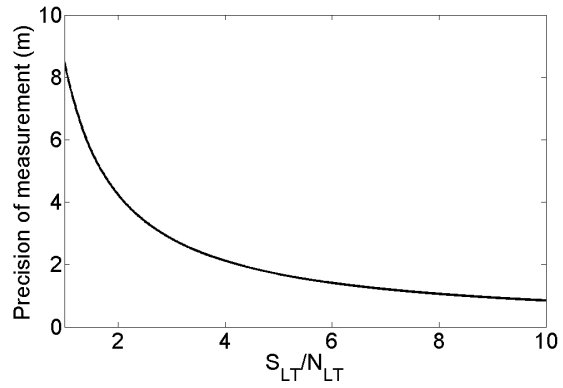


Fig. 6. Precision as function of  $S_{LT}/N_{LT}$

In the case of considering a finite receiver bandwidth the measurement precision would be much higher than the estimation provided by (12).

## 6. – Conclusions

The theoretical analysis of the performance of a RM-CW LIDAR system has allowed the determination of the signal to noise ratio, the probability of catastrophic errors due to random noise peaks, and the precision in the distance measurement. The analytical expressions derived are a valuable tool for the design and the evaluation of the performance of these systems.

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